Combinatorics

Final Examination

Instructions: All questions carry equal marks.

1. Show that a $k \times n$ Latin rectangle with $k < n$ can be extended to a Latin square of order n.

2. Suppose that C is a binary code, not necessarily linear, with length 23, minimum distance 7 and having 2^{12} codewords. Assume that the zero vector belongs to C. Then, show that there are eactly 253 words of weight 7.

3. Prove that in a Steiner system $S(2, k, v)$ with $b > v$, we must have $v \geq k^2$. Further prove that, in case $v = k^2$, the set of blocks can be partitioned into $k+1$ "parallel classes" of k blocks such that the blocks in a given parallel class are pairwise disjoint.

4. Let a, d be natural numbers. If $b = \lceil \frac{a}{d} \rceil$ or $\lfloor \frac{a}{d} \rfloor$, then prove that

$$\lceil \frac{a-b}{d-1} \rceil \leq \lceil \frac{a}{d} \rceil \quad \text{and} \quad \lfloor \frac{a-b}{d-1} \rfloor \geq \lfloor \frac{a}{d} \rfloor.$$

5. Define combinatorial geometry and modular combinatorial geometry. Prove that a combinatorial geometry is modular if and only if any line and any hyperplane meet nontrivially.